(2p)

Turma Y

Pr.1

0,3 $\eta_1 := \begin{bmatrix}1\\0\\0\end{bmatrix}$, $|\eta_1|^2 = 1$

0,7 $\eta_2 := \overset{\xi_2}{\begin{bmatrix}1\\2\\1\end{bmatrix}} - \frac{\langle \overset{\eta_1}{\begin{bmatrix}1\\0\\0\end{bmatrix}}, \overset{\xi_2}{\begin{bmatrix}1\\2\\1\end{bmatrix}} \rangle}{\underbrace{|\eta_1|^2}_{=1}} \begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}0\\2\\1\end{bmatrix}$, $|\eta_2|^2 = 5$

1 $\eta_3 := \overset{\xi_3}{\begin{bmatrix}0\\1\\-3\end{bmatrix}} - \underbrace{\cancel{\frac{\langle \overset{\eta_1}{\begin{bmatrix}1\\0\\0\end{bmatrix}}, \overset{\xi_3}{\begin{bmatrix}0\\1\\-3\end{bmatrix}} \rangle}{\underbrace{|\eta_1|^2}_{1}}}}_{0} \overset{\eta_1}{\begin{bmatrix}1\\0\\0\end{bmatrix}} - \underbrace{\frac{\langle \overset{\eta_2}{\begin{bmatrix}0\\2\\1\end{bmatrix}}, \overset{\xi_3}{\begin{bmatrix}0\\1\\-3\end{bmatrix}} \rangle}{\underbrace{|\eta_2|^2}_{5}}}_{= -1/5} \overset{\eta_2}{\begin{bmatrix}0\\2\\1\end{bmatrix}} = \begin{bmatrix}0\\7/5\\-14/5\end{bmatrix} = \frac{1}{5}\begin{bmatrix}0\\7\\-14\end{bmatrix}$

checar se calculamos certo

$\eta_1 \perp \eta_2$: $\begin{bmatrix}1\\0\\0\end{bmatrix} \perp \begin{bmatrix}0\\2\\1\end{bmatrix}$ ✓

$\eta_1 \perp \eta_3$: $\begin{bmatrix}1\\0\\0\end{bmatrix} \perp \begin{bmatrix}0\\1\\-3\end{bmatrix}$ ✓

$\eta_2 \perp \eta_3$: $\langle \begin{bmatrix}0\\2\\1\end{bmatrix}, \frac{1}{5}\begin{bmatrix}0\\7\\-14\end{bmatrix} \rangle = \frac{1}{5}(2\cdot 7 - 14) = 0$ ✓

(Turma Y)

Pr. 2 2 $F := \{p \in E := P_3 \mid p(-1) = p(1) = 0\}$

1 (a) base ortog. de F: $p \in F$
$p(x) = a + bx + cx^2 + dx^3$

$p(1) = p(-1) = 0 \iff \begin{cases} a+b+c+d=0 \\ a-b+c-d=0 \end{cases} \iff a = -c$ e $b = -d$

0,4

$\to p(x) = a(1-x^2) + b(x-x^3)$

$\to \boxed{F \text{ gerado por } \{1-x^2, x-x^3\} =: B}$ 0,1

ortogonal? $\langle 1-x^2, x-x^3\rangle = \int_{-1}^{1} x - 2x^3 + x^5 \, dx = 0$ ✓. 0,4

$\Rightarrow B$ ortogonal $\overset{\text{nenhum}}{\underset{\text{elemento } 0}{\Longrightarrow}}$ B LI $\Rightarrow B$ base ortogonal 0,1

1 (b) $q = 1 + 2x - x^3$ queremos $f \in F, g \in F^\perp$ tq $q = f + g$

$P_3(\mathbb{R}) = F \oplus F^\perp$
$\underset{\text{dado}}{q} = f + g$

(i) $f := pr_F\, q$
(ii) $g := q - r$

(i) $f = pr_F\, q = \frac{\langle q, 1-x^2\rangle}{\|1-x^2\|^2}(1-x^2) + \frac{\langle q, x-x^3\rangle}{\|x-x^3\|^2}(x-x^3)$

$\langle 1+2x-x^2, 1-x^2\rangle = \frac{16}{15}$, $\langle 1+2x-x^2, x-x^3\rangle = \frac{1}{3}$

$\|1-x^2\|^2 = \frac{16}{15}$, $\|x-x^3\|^2 = \frac{2}{3}$

$\Rightarrow \boxed{f(x) = 1 - x^2 + \frac{1}{2}(x-x^3)}$ $\Rightarrow \boxed{g(x) = \frac{1}{2}(3x - x^3)}$

0,75 $\quad := q - r$ $\quad$ 0,25

Pr. 3 (2p) (Turma Y)

0,25 $b := [B]_{\mathcal{E},\mathcal{E}} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$\boxed{\begin{array}{l} ae^{11}+be^{12}+ce^{21}+de^{22} \\ \overset{A}{\longmapsto}(a+b)e_1+(b+c)e_2 \\ \qquad +(c+d)e_3 \end{array}}$

0,75 $a := [A]_{\mathcal{B},\mathcal{E}} : \mathbb{R}^4 \longrightarrow \mathbb{R}^3$

$(\mathbb{R}\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad \begin{cases} A e^{11} = e_1 \cdot \overset{a}{1} \\ A e^{12} = e_1 \cdot \overset{b}{1} + e_2 \cdot \overset{b}{1} \\ A e^{21} = e_2 \cdot \overset{c}{1} + e_3 \cdot \overset{c}{1} \\ A e^{22} = e_3 \cdot \overset{d}{1} \end{cases}$$

$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \overset{b}{1} \cdot \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

BA   Tem. aula

$c := [C]_{\mathcal{B},\mathcal{E}} \overset{0,5}{=} [B]_{\mathcal{E},\mathcal{E}} [A]_{\mathcal{B},\mathcal{E}} = b\,a$

$= \begin{bmatrix} 1 & -1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 2 & 1 & 0 \end{bmatrix}$ 0,5

Pr.4 (2p) $\dim E = n < \infty$, $\langle \cdot,\cdot \rangle$ (Turma Y)

0,5 (a) FALSO: $\{v, 0\}$ ortogonal: $\langle v, 0\rangle = 0$ ✓

0,5 (b) VERDADEIRO: para $v, w \in F^\perp$, $\alpha, \beta \in \mathbb{R}$, $u \in F$

$$\langle \alpha v + \beta w, u \rangle = \alpha \underbrace{\langle v, u\rangle}_{=0} + \beta \underbrace{\langle w, u \rangle}_{=0} = 0$$

$\Rightarrow \alpha v + \beta w \in F^\perp \Rightarrow$ fechado sob $+, \cdot$

$\Rightarrow$ subesp.

0,5 (c) FALSO: $\dim E = \dim N(C) + \overbrace{\dim Im(C)}^{= \dim N(C) \text{ hip.}}$

$= 2 \dim N(C)$ é par, não ímpar.

0,5 (d) VERDADEIRO: Seja $\{\xi_1, \dots, \xi_n\}$ LI

suponha $0 = \alpha_1 D\xi_1 + \dots + \alpha_n D\xi_n$

$\underset{D \text{ lin.}}{=} D(\alpha_1 \xi_1 + \dots + \alpha_n \xi_n)$

$\overset{D \text{ inj.}}{\Longrightarrow} \alpha_1 \xi_1 + \dots + \alpha_n \xi_n = 0$

$\underset{LI}{\overset{\{\xi_1, \dots, \xi_n\}}{\Longrightarrow}} \alpha_1 = \dots = \alpha_n = 0$

#

(RP) $A: \mathbb{R}^4 \to \mathcal{P}_3(\mathbb{R}) \cong \mathbb{R}^4$ $\boxed{\text{Turma Y}}$

Pr. 5

Seja $N := \left\langle \begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \begin{pmatrix}1\\0\\1\\0\end{pmatrix} \right\rangle \subset \mathbb{R}^4$

LI $\Leftrightarrow$ $\boxed{\dim N = 2}$

$$\Bigg\{ \underbrace{\begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \begin{pmatrix}1\\0\\1\\0\end{pmatrix}}_{\text{dados}}, \begin{pmatrix}0\\0\\0\\1\end{pmatrix}, \begin{pmatrix}1\\0\\-1\\0\end{pmatrix} \Bigg\}$$ conjunto ortogonal

óbviamente $\perp$ a todos; $\perp$ a todos

normalizar:

$$\underbrace{\varepsilon_1 = \begin{pmatrix}0\\1\\0\\0\end{pmatrix}, \varepsilon_2 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\1\\0\end{pmatrix}}_{\text{base de } N}, \underbrace{\varepsilon_3 = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}, \varepsilon_4 = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\0\\-1\\0\end{pmatrix}}_{\text{base de } N^\perp}$$

$B: \mathbb{R}^4 \to \mathbb{R}^4$

$$v \mapsto pr_{N^\perp} v := \underbrace{\langle \varepsilon_3, v\rangle}_{d} \varepsilon_3 + \underbrace{\langle \varepsilon_4, v\rangle}_{\frac{1}{\sqrt{2}}(a-c)} \varepsilon_4$$

$v = \begin{pmatrix}a\\b\\c\\d\end{pmatrix}$

$$= \begin{pmatrix}0\\0\\0\\d\end{pmatrix} + \frac{1}{\sqrt{2}\cdot\sqrt{2}}\begin{pmatrix}a-c\\0\\c-a\\0\end{pmatrix} = \boxed{\begin{pmatrix}\frac{a-c}{2}\\0\\\frac{c-a}{2}\\d\end{pmatrix}} =: Bv$$

checar núcleo $B\begin{pmatrix}0\\1\\0\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\end{pmatrix}$ ✓

$B\begin{pmatrix}1\\0\\1\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\\0\end{pmatrix}$ ✓

$\mathbb{R}^4 \simeq \mathcal{P}_3(\mathbb{R})$

$e_1, e_2, e_3, e_4 \leftrightarrow 1, x, x^2, x^3$

$$\Rightarrow \boxed{A\begin{pmatrix}a\\b\\c\\d\end{pmatrix} = \frac{a-c}{2} + \frac{c-a}{2}x^2 + dx^3}$$

isso é uma solução

tem muitos outros...