Turma E

Pr. 1 $(\mathbb{R}^3, \langle \cdot , \cdot \rangle_0)$, com GS ortogonalize $\mathcal{B} = \{\xi_1, \xi_2, \xi_3\}$

$$\xi_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \xi_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \quad \xi_3 = \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}.$$

$$\eta_1 := \xi_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad |\eta_1|^2 = 0^2 + 0^2 + 1^2 = 1$$

$$\eta_2 := \xi_2 - pr_{\eta_1} \xi_2 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} - \frac{\overbrace{\left\langle \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\rangle}^{=1}}{\underbrace{|\eta_1|^2}_{=1}} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad |\eta_2|^2 = 1 + 2^2 = 5$$

$$\eta_3 := \xi_3 - pr_{\eta_1} \xi_3 - pr_{\eta_2} \xi_3$$

$$= \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix} - \cancel{\frac{\overbrace{\left\langle \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix} \right\rangle}^{0}}{|\eta_1|^2}} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \overset{=+1}{-} \frac{\overbrace{\left\langle \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix} \right\rangle}^{1-6=-5}}{\underbrace{|\eta_2|^2}_{5}} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$$

$$\Rightarrow \{\eta_1, \eta_2, \eta_3\} = \left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} \right\} \text{ base ortogonal}$$

Turma E

Pr. 2 $\langle \cdot, \begin{bmatrix} 2&0&0\\0&3&0\\0&0&1\end{bmatrix} \cdot \rangle_{\underset{\text{euclidiano}}{0}} =: \langle \cdot, \cdot \rangle$

(a) $F = \{ \underbrace{x - y + z = 0}_{\substack{\text{SLH 1 eq.} \\ \text{a 3 incógn.}}} \} \ni \underbrace{\begin{pmatrix}1\\1\\0\end{pmatrix}}_{\xi_1}, \underbrace{\begin{pmatrix}0\\1\\1\end{pmatrix}}_{\xi_2}$

$\rightsquigarrow \dim F = 2$

LI $\begin{pmatrix}0\\0\\0\end{pmatrix} = \alpha \begin{pmatrix}1\\1\\0\end{pmatrix} + \beta \begin{pmatrix}0\\1\\1\end{pmatrix} = \begin{pmatrix}\alpha\\ \beta\\ \beta\end{pmatrix}$

$\Rightarrow \alpha = \beta = 0$ ✓

$\Downarrow$

gera $\Rightarrow \{\xi_1, \xi_2\}$ base de F.

- $\eta_1 := \xi_1 = \begin{pmatrix}1\\1\\0\end{pmatrix}$, $|\eta_1|^2 = \langle \begin{pmatrix}1\\1\\0\end{pmatrix}, \underbrace{\begin{bmatrix}2&&\\&3&\\&&1\end{bmatrix} \begin{pmatrix}1\\1\\0\end{pmatrix}}_{\begin{pmatrix}2\\3\\0\end{pmatrix}} \rangle_0 = 2+3 = 5$

- $\eta_2 := \underset{\xi_2}{\begin{pmatrix}0\\1\\1\end{pmatrix}} - \dfrac{\langle \begin{pmatrix}1\\1\\0\end{pmatrix}, \overbrace{\begin{bmatrix}2&&\\&3&\\&&1\end{bmatrix} \begin{pmatrix}0\\1\\1\end{pmatrix}}^{\begin{pmatrix}0\\3\\1\end{pmatrix}} \rangle_0}{\underbrace{|\eta_1|^2}_{=5}} \begin{pmatrix}1\\1\\0\end{pmatrix} = \begin{pmatrix}0\\1\\1\end{pmatrix} - \frac{3}{5}\begin{pmatrix}1\\1\\0\end{pmatrix} = \frac{1}{5}\begin{pmatrix}-3\\2\\5\end{pmatrix}$

$= 0+3+0 = 3$

$|\eta_2|^2 = \frac{1}{25} \langle \begin{pmatrix}-3\\2\\5\end{pmatrix}, \begin{bmatrix}2&&\\&3&\\&&1\end{bmatrix}\begin{pmatrix}-3\\2\\5\end{pmatrix} \rangle$

$= \frac{18+12+25}{25} = \frac{55}{25} = \frac{11}{5} \to$ (b)

$\underline{\eta_1} = \xi_1 \underline{\in F}$ ✓

$\eta_2 = \frac{1}{5}\begin{pmatrix}-3\\2\\5\end{pmatrix}$: $\frac{1}{5}(\overset{x}{-3} \overset{-y}{-2} \overset{+z}{+5}) = 0$ ✓

$\Rightarrow \underline{\eta_2 \in F}$

$\underline{\eta_1 \perp \eta_2}$ $\quad \langle \eta_1, \eta_2 \rangle = \langle \begin{pmatrix}1\\1\\0\end{pmatrix}, \underbrace{\begin{bmatrix}2&&\\&3&\\&&1\end{bmatrix} \begin{pmatrix}-3\\2\\5\end{pmatrix}\frac{1}{5}}_{\begin{pmatrix}-6\\6\\5\end{pmatrix}\cdot\frac{1}{5}} \rangle_0 = \frac{1}{5}(-6+6+0)$

$= 0$ ✓

$\Downarrow$

$\boxed{\underset{\neq 0}{\eta_1} \perp \underset{\neq 0}{\eta_2}}$

$\Rightarrow \{\eta_1, \eta_2\}$ LI

$\Rightarrow \{\eta_1, \eta_2\}$ base ortogonal de F

(b)

$$pr_F v = \frac{\langle \eta_1, v \rangle}{\underbrace{|\eta_1|^2}_{\overset{(a)}{=} 5}} \eta_1 + \frac{\langle \eta_2, v \rangle}{\underbrace{|\eta_2|^2}_{\overset{(a)}{=} 11/5}} \eta_2$$

$$= \frac{\overbrace{\left\langle \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 & & \\ & 3 & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\rangle}^{2+6+0}}{5} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \frac{\overbrace{\left\langle \frac{1}{5}\begin{pmatrix} -3 \\ 2 \\ 5 \end{pmatrix}, \begin{pmatrix} 2 & & \\ & 3 & \\ & & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \right\rangle}^{(-6+12+15)/5 = 21/5}}{11/5} \frac{1}{5} \begin{pmatrix} -3 \\ 2 \\ 5 \end{pmatrix}$$

$$= \frac{8}{5} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \frac{21}{11 \cdot 5} \begin{pmatrix} -3 \\ 2 \\ 5 \end{pmatrix}$$

$$= \frac{88}{55} \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \begin{pmatrix} -63/55 \\ 42/55 \\ 21/11 \end{pmatrix} = \begin{pmatrix} 5/11 \\ 26/11 \\ 21/11 \end{pmatrix} = \frac{1}{11} \begin{pmatrix} 5 \\ 26 \\ 21 \end{pmatrix}$$

Turma E

pr.3

$$a := [A]_{\mathcal{E},\mathcal{E}} = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}$$

$$b := [B]_{\mathcal{E},\mathcal{B}} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$c := [\underbrace{C}_{=BA}]_{\mathcal{E},\mathcal{B}} \overset{\text{Teor. aula}}{=} [B]_{\mathcal{E},\mathcal{B}} [A]_{\mathcal{E},\mathcal{E}}$$

$$= \begin{bmatrix} 1 & -1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} -1 & -5 \\ 2 & 3 \\ 3 & 1 \end{bmatrix}$$

Turma E

Pr. 4 $\dim V = 200$

Teor. núcl. & im.

$\Downarrow$ $\dim V = 200$

(a) FALSO: $3500 = 20(\dim N(A) + \dim \mathrm{Im}(A)) + 10 \dim N(A)$

$= 4000 + 10 \cdot \underbrace{\dim N(A)}_{\geq 0} \geq 4000$

contradição

(c) FALSO: suponha por absurdo $\mathrm{Im}(B) \subset N(B)$,

$\Rightarrow \underset{200}{\underbrace{\dim V}} = \dim N(B) + \underbrace{\dim \mathrm{Im}(B)}_{=150}$

$\Rightarrow 50 = 200 - 150 = \dim N(B) \geq \dim \mathrm{Im}(B) = 150$

(contradição

(b) FALSO: contraexemplo com $\mathrm{Im}\, C \not\subset N(C)$

é assim:

Seja $F \subset V$ subesp. de dim 185

e $G \subset V$ " " " 15

t.q. $F \oplus G = V$

$\boxed{C := P_{F,G}} \Rightarrow \mathrm{Im}(C) = F$

$N(C) = G$

projeção

sobre F paralelamente G

185	$\not\subset$ 15	dim
F	$\not\subset G$	

existência de G: escolha prod. interno em V

$G := F^\perp$ (existe segundo aula)

Turma E

Pr. 5

$$A : \underset{\mathbb{R}^4}{P_3} \longrightarrow \underset{\mathbb{R}^3}{P_2} \qquad \underline{a} := [A]_{\mathcal{E}_{P_3}, \mathcal{E}_{P_2}}$$

$\mathcal{E}_{P_3} = \{1, x, x^2, x^3\}$ $\mathcal{E}_{P_2} = \{1, x, x^2\}$

$\mathbb{R} \times 0s$ = pols. const.

$$\mathbb{R}^4 \xrightarrow{\underline{a}} \mathbb{R}^3$$

$$\boxed{\mathrm{Im}\, A = 0 \times \mathbb{R}^2}$$

$$\underline{a} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ e & f & e & -e \\ i & j & i & -i \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \\ \delta \end{bmatrix}$$

$$N(A) = \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \right\rangle$$

LI

$\Downarrow$

$\boxed{\dim N(A) = 2}$

$\Rightarrow \boxed{\dim \mathrm{Im}\, A = 2}$

colunas geram Im(A)

$$\left\langle \begin{pmatrix} 0 \\ 1 = e \\ 0 = i \end{pmatrix}, \begin{pmatrix} 0 \\ 0 = f \\ 1 = j \end{pmatrix} \right\rangle = 0 \times \mathbb{R}^2$$

$\rightsquigarrow$ escolhemos $\boxed{\begin{matrix} e = 1, & j = 1 \\ i = 0, & f = 0 \end{matrix}}$

$$\underline{a} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \text{ posto } 2$$

$\mathrm{Im}\, A = 0 \times \mathbb{R}^2$ ✓

$N(A) = \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \right\rangle$ ✓

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} 0 \\ a + c - d \\ b \end{bmatrix}$$

$$\boxed{a + bx + cx^2 + dx^3 \xmapsto{A} (a + c - d)x + bx^2}$$