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Análise Real II

Teoria de medida e integração

Lista 8 – Weak derivatives

Exercise 1. Let $I = (-1, +1)$. For any $p \in [1, \infty]$ show the following:

(i) The absolute value function $u(x) = |x|$ is weakly differentiable on I. A weak derivative u_1 is given by the **sign function**

$$\operatorname{sign}(x) := \begin{cases} -1 & , x < 0, \\ 0 & , x = 0, \\ +1 & , x > 0. \end{cases}$$

(ii) Is (i) still true when I is replaced by the domain $\mathbb{R}$?

(iii) The sign function on I does not have a weak derivative. What happens on $\mathbb{R}$?

Exercise 2. Note that for weak differentiability the *corner* of $|\cdot|$ did not matter, but the *jump discontinuity* of sign was a problem. Check that sign is not absolutely continuous. Is the Cantor function c weakly differentiable? What is the problem?

Exercise 3. Let $Q \subset \mathbb{R}^2$ be the open unit ball and consider the function $u(x) := |x|^{-\gamma}$ for $x \neq 0$ and $u(0) := 0$. For which values of $\gamma > 0$ and $p \in [1, \infty]$

- is $u \in \mathcal{L}^p(Q)$?
- does u admit a weak derivative?

Exercise 4. Let $Q \subset \mathbb{R}^2$ be the open unit ball. Find an example of $p \geq 1$ and a $W^{1,p}$ function $u : Q \to \mathbb{R}$ which is unbounded with respect to the L^∞-norm on each open subset U of Q. Conclude that your example is not Lipschitz continuous.